Physics 4A Notes

SCIENCE

Science

Science is a human activity with the function of discovering the orderliness of nature and finding the causes that govern this order.

Science includes the body of knowledge about nature that represents the collective efforts, insights, findings and wisdom of the human race.

Fact
something known to have occurred or to be true

Theory
A set of propositions describing the operation and causes of natural phenomena

Hypothesis
A tentative assertion about natural phenomena, assumed but not positively known

Scientific Method
1. Recognize the problem
2. Develop a **hypothesis** (guess an answer)
3. Predict the consequences of hypothesis
4. Perform experiment to test predictions
5. Formulate the simplest **theory** that organizes the three main ingredients: hypothesis, prediction, experimental outcome

Scientific Claim
Can be tested; can be proved wrong with experiment

Which of the following is a scientific claim?
a. the moon is made of green cheese
b. intelligent life likely exists in the universe
c. Albert Einstein is the greatest physicist so far in the twentieth century

PHYSICS - WHAT IS IT?

Physics - (Dicta.) science of matter and energy, and the interaction between the two.

Physics is concerned with the description of nature.
Gives "rules" of nature. Concerned with how things work.

Topics in physics include acoustics, optics, mechanics, thermodynamics (heat), electricity, magnetism, the atom, the nucleus, and relativity.

Applications of physics can be found in medicine, dentistry, agriculture, architecture, home economics, nursing, geology, astronomy, engineering, technical fields, computer fields, sports, driving your car, cooking, cleaning, weather, ecology, construction, health, toys, breathing...
everything in your universe.

Limits.
Physics cannot answer many of the questions from topics dealing with religion, ethics, psychology or philosophy.

PHYSICAL QUANTITIES

One of the questions in physics is how do we describe nature?

Measurement: an act of comparing an object to a standard.

Physics is a quantitative science. **It measures nature.**

Fundamental units: standard units.
Derived units: made up of one or more fundamental units.

Fundamental Units and the SI (System International)

1. length	meter
2. mass	kilogram
3. time	second
4. current	ampere
5. temperature	Kelvin
6. amount of substance	mole
7. luminous intensity	candela

mks is a subset of the SI system of units.
cgs
ft-lb-s

FUNDAMENTAL UNITS	DERIVED UNITS
Length L	Area
Time T	Volume
Mass M	Speed
Temperature	Velocity
Electric Current	Acceleration
Luminous Intensity	Force
Number of Particles	Momentum
	Energy

THE CONVERSION OF UNITS

The changing of quantities from one unit to another is a necessary but often tedious task. The table of conversion factors is found in the front cover of your text. The task of unit conversion can often be made less messy by carrying out the conversion as a product with the conversion factors and their associated units treated like numerical fractions.

Example: Convert 12 inches to centimeters.

Solution:

1. Write an equivalent statement:

   1 inch = 2.54 cm

2. Form the two possible ratios from the equivalent statement. These ratios are equal to one.

   $$\frac{1 \text{ inch}}{2.54 \text{ cm}} \quad \text{or} \quad \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

3. Multiply the number to be converted with the ratio that has the desired unit in the numerator. In this case the desired unit is cm.

   $$(12 \text{ inches})\frac{(2.54 \text{ cm})}{(1 \text{ inch})} = 30.5 \text{ cm}$$

Any number of such products can be strung together to get the desired final units.

.0030 km	1000 m	100 cm	1 cm	=	**1181 cm**
	1 km	1m	2.54 in		

ACCURACY VS. PRECISION

Accuracy is a measure of how close a measurement is to the accepted value. For example, when measuring the acceleration due to gravity which has an accepted value of 9.80 m/s^2, 9.7 m/s^2 is more accurate than 12.01 m/s^2.

Precision is a measure of the agreement between a set of measurements. For example, the set of measurements 12.00 m/s^2, 12.01 m/s^2, 12.02 m/s^2 is more precise than the set of measurements 9.6 m/s^2, 9.7 m/s^2, and 9.8 m/s^2.

ERRORS

If we knew that every measurement we made was absolutely accurate, experimentation would be greatly simplified. Whenever we read a meter stick, or see a needle point to a number, there are several types of errors that can be made.

1. **Personal or Human Error**: We read an instrument incorrectly or make the data fit what we think the results should be.

2. **Instrumental Errors**: These are the *built-in errors of the instrument*. For example, a voltmeter may be accurate to 4 percent, or a micrometer may have a screw of unequal pitch.

3. **Systematic Error**: Errors in one direction or errors *that affect all measurements the same way.* They are caused by such things as not zeroing an instrument properly. Personal and instrument errors can be systematic errors.

4. **Random Errors**: Random errors are *deviations that occur in an experiment that are beyond the experimenter's control.* If there are many data points measuring one quantity, these measurements will be distributed over a range. For example, in measuring one quantity, these measurements will be distributed over a range. For example, in measuring a table top, we may get ten readings of 50.01 cm, five readings of 50.02 cm, five readings of 50.00 cm, two readings 50.03 cm, one reading of 49.9 cm and so on. The experimenter's problem is to establish the "best" value of the measurement. The average or arithmetic mean value is usually accepted as the most probable value of the measurement.

Percent error is used to get a feel for the accuracy of a measurement. It is used when comparing an experimental value to an accepted value.

$$\%\ error = \frac{|\text{experimental value} - \text{accepted value}|}{\text{accepted value}}\ x\ 100\%$$

$$\%\ error = \frac{|E - A|}{A}\ x\ 100\%$$

Percent difference is used to get a feel for the precision of two measurements.

$$\%\ difference = \frac{|\text{measurment\#1} - \text{measurement\#2}|}{\text{average of measurements \#1 and \#2}}$$

$$\%\ difference = \frac{|E_1 - E_2|}{\frac{E_1 + E_2}{2}}\ x\ 100\%$$

SIGNIFICANT FIGURES

I. DEFINITIONS OF SIGNIFICANT FIGURES

All the numbers in an experimental measurement that can be read directly from the instrument plus one doubtful or estimated number.

II. RULES FOR WRITING NUMBERS USING THE CONCEPT OF SF

1. All non-zero digits are counted as SF.

2. Zeros embedded between non-zero digits are counted as SF; e.g. 1.0003 has 5 SF.

3. Zeros written to the left of other digits are not counted as SF; e.g. 0.00000002 has 1 SF.

4. If there is no decimal point, the right-most non-zero digit is the least significant; e.g. 240 has 2 SF.

5. If there is a decimal point, the right-most digit is the least significant, even if it is zero; e.g. 240. has 3 SF. 240.0 has 4 SF.

6. Writing the number in scientific notation will eliminate confusion about rules 3 and 4; e.g.

2×10^{-8} has only 1 SF	2.00×10^{-8} has 3 SF
2.5000×10^{4} has 5 SF	2.5×10^{4} has 2 SF

III. THE RESULT RULE FOR MULTIPLICATION AND DIVISION WITH SF

The number of significant figures in the result equals the number of significant figures in the measurement with the least number of significant figures.

$$\begin{array}{r} 6.27 \\ \underline{\times\ 5.5} \\ 34.485 \end{array} \Rightarrow \mathit{34} \qquad\qquad \frac{234}{22} = 10.636 \Rightarrow \mathit{11}$$

IV. THE RESULT OF ADDITION AND SUBTRACTION

In addition and subtraction, adding or subtracting begins with the first column from the left that contains a doubtful figure. The numbers are rounded off to this column and all digits to the right are dropped.

$$\begin{array}{r} 42.31 \\ 0.0621 \\ 512.4 \\ \underline{2.57} \end{array} \qquad\qquad \begin{array}{r} 42.3 \\ 0.1 \\ 512.4 \\ \underline{2.6} \\ \mathit{557.4} \end{array}$$

V. ROUNDING OFF

A number is rounded off to the desired number of SF by dropping one or more digits to the right. When the first digit dropped is less than five, the last digit retained should remains unchanged. When it is five or more, increase the last digit by one.

Significant figures in angles

Accuracy in sides	θ in decimals
1 SF	10's
2SF	whole
2 SF	tenth
3SF	hundredth
4SF	thousandth

ABSOLUTE AND PERCENTAGE ERRORS

Example of absolute error: 97 ± 2 cm ; 12 ± 2 cm; 1.03 s ± 0.01 s; 104.89 ± 0.01 s
Examples of percentage error: 97 ± 2% cm; 12 ± 20%; 1.03 ± 1%; 104.89 ± 0.01%

To find compound error in addition and subtraction, *add* the *absolute* error.

(97 ± 2 cm) + (12 ± 2 cm) = (109 ± 4 cm)
(104.89 ± 0.01 s) – (1.03 s ± 0.01 s) = 103.86 ± 0.02 s
(88 ± 2 kg) + (3.26 ± 0.02 kg) = (91 ± 2 kg)

To find the compound error in multiplication or division, *add the percentage* error.

Area of square with sides 97 cm ± 2 % = 9400 cm^2 ± 4 % = 9400 ± 400 cm^2

(97 cm ± 2 %) x (12 cm ± 20 %) = 1200 cm^2 ± 20 % = 1200 ± 200 cm^2

(104.89 s ± 0.01%)(1.03 s ± 1 %) = 100 ± 1 % = 100 ± 1

True area = LW

Calculated area = (L + l)(W + w) = LW +lW +wL +lw

% error in area = [(calculated area – true error) / true area] * 100

$$\frac{lW + wL + lw}{LW} * 100 = \frac{l}{L}100 + \frac{w}{W}100 + \frac{lw}{LW}100 =$$

= (% error in L) (% error in W) + (comparatively small term)

GRAPHS

A scientific experiment usually involves changing one physical quantity and observing the result on a second physical quantity. The physical quantity, which is changed by the experimenter, is called the ***independent*** variable while the one, which responds to the change, is called the ***dependent*** variable. For example, if the experimenter were to drop a ball from different heights and observe the velocity when the ball hits the ground, the independent variable would be the heights, and the dependent variable would be the velocities. The measurements made are displayed in the form of a table.

Usually, it is easier to determine the relationship between the variables visually in the form of a graph rather than from a table. A graph is a pictorial representation of these ordered pairs of numbers from which a person may:

a. Visually determine how the dependent quantity changes as a function of the independent quantity.
b. Calculate the relationship between the quantities.

When plotting a graph, usually the dependent variable is plotted on the ordinate or y-axis while the independent variable is plotted on the abscissa or x-axis. It is standard to name the graph "y" vs. "x" and then label the axes with the proper units.

After the ordered pairs of numbers are plotted on graph paper, the plotted points are connected by a smooth curve. The curve that is drawn may be a straight line or it may be curved. The general name "curve" is used in reference to all graphs whether the line is actually curved or straight.

Many of the relationships studied in Engineering Physics are linear:

$$y = f(x) = mx + b$$

The graph of such a function is a straight line. Often it will be of interest to calculate the slope *m* of that line to experimentally determine the value of some physical constant.

The slope of a straight line is defined as the measure of the rise of the line compared to the run of the line, or change in the *y*-values divided by the corresponding change in the *x*-values. The usual symbol for slope is *m*.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

CALCULUS

Function – Relation between two terms called variables because their values vary.

Definition of Derivative of f(x).

Definition of derivative: $f'(x) = \lim \Delta x \rightarrow 0 \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$

If *y=f(x), the dy=f'(x)dx* or *(dy/dx) = f'(x)*

Indefinite integral (also know as antiderivative).

$\int f(x)dx = F(x) + c$ where "c" is constant of integration also know as integrand. "x" is the variable of integration in this example.

if and only if *F'(x)=f(x)*
F(x) is also known as the antiderivative (reverse of differentiating) of *f(x)*

Definite integral of *f(x)* from *a* to *b*:

$$\int_a^b f(x)dx = \lim \Delta x \rightarrow 0 \sum_i f(x_i)\Delta x$$

<u>Fundamental Theorem of Calculus:</u>

f(x) = F'(x), then F(x_2)-F(x_1) = $\int_{xt_1}^{x_2} f(x)dx$

Relationship between definite and indefinite integrals

$$\int_a^b f(x)dx = \left[\int f(x)dx \right]_a^b$$

MOTION (Conceptual)

MOTION IS A CHANGING OF POSITION.

I. **Scalar** - Magnitude
Examples: volume, speed, length, mass

II. **Vector** - Magnitude and Direction

III. **Speed** - How fast an object is traveling (scalar)
Instantaneous Speed - speed of an object at an instant in time
Constant or Uniform Speed - speed doesn't change

IV. **Velocity** - Speed and Direction (vector)
Instantaneous velocity - velocity of an object at an instant in time

V. **Acceleration** = change in velocity
Instantaneous Acceleration - acceleration of an object at an instant in time

MOTION (Mathematical)

Displacement: difference between the final position and the initial position.
We represent displacement with Δx, which is read "delta x".
Vector

$$\Delta x = x_f - x_o$$

Average velocity: ratio of displacement to change in time for that displacement.
Vector

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_0}$$

Average speed = *(total distance traveled) / Δt*
Scalar

Average acceleration:
Vector.

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_0}$$

CALCULUS AND MOTION

Instantaneous velocity:

$$v(t) = \frac{dx(t)}{dt}$$

The instantaneous velocity is the derivative with respect to time of the position function.

Instantaneous acceleration:

$$a(t) = \frac{dv(t)}{dt}$$

The instantaneous acceleration is the derivative with respect to time of the velocity function.

Graphical Interpretation of Position, Velocity and Acceleration

The **average velocity** between two points in time can be obtained from a position-versus-time graph by computing the slope of the straight line joining the coordinates of the two points.

The **instantaneous velocity** at time *t* is the slope of the tangent line drawn to the position-versus-time graph at that time.

The **average acceleration** between two points in time can be obtained from a velocity-versus-time graph by computing the slope of the straight line joining the coordinates of the two points.

The **instantaneous acceleration** at time *t* is the slope of the tangent line drawn to the velocity-versus-time graph at that time.

The change in displacement between two points can be found by finding the area under the velocity vs. time graph between those two points. **Integration.** The change in velocity between two points can be found by finding the area under the acceleration vs. time graph between those two points.

Integration. Exercise:
Using integration, show that if the acceleration is constant, then

$$x = x_i + v_i t + \frac{1}{2}at^2$$

CONSTANT ACCELERATION

Use the formulas below for *constant acceleration* and *straight-line motion* only.

$$v = v_0 + at$$
$$\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$\vec{x} - \vec{x}_0 = \vec{v}_{avg}t = \left(\frac{\vec{v}_0 + \vec{v}}{2}\right)t$$
$$v^2 = v_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

t = time; ***v*** = instantaneous velocity at time t; ***v***$_o$ = initial velocity
a = acceleration; ***x*** = final displacement; ***x***$_o$ = initial displacement

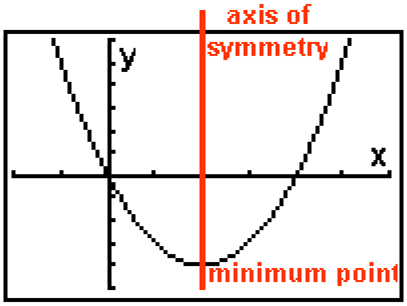
Be careful of signs!!

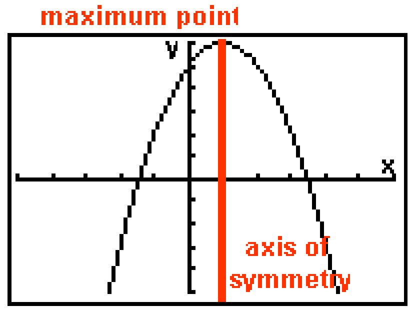
If car speeding up, **v** and **a** in same direction.
If car slowing down, **v** and **a** in opposite direction

FREE-FALL WHEN V$_o$ = 0

t(s)	***a(m/s^2)***	***v = at(m/s)***	***d = 1/2 at^2(m)***
0	**10**	**0**	**0**
1	**10**	**10**	**5**
2	**10**	**20**	**20**
3	**10**	**30**	**45**
4	**10**	**40**	**80**
5	**10**	**50**	**125**
.	**.**	**.**	**.**
.	**.**	**.**	**.**
t	**10**	**10t**	**.5t^2**

LAW OF FALLING BODIES
In a vacuum, all objects fall with the same constant acceleration, g.





If **a** is positive, the parabola opens upward and has a minimum point.
The axis of symmetry is
x = (-b)/2a

If **a** is negative, the parabola opens downward and has a maximum point.
The axis of symmetry is
x = (-b)/2a.

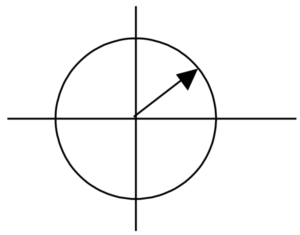
Parabolas are of the form: $\mathbf{y} = a\mathbf{x}^2 + b\mathbf{x} + c$

TRIGONOMETRY

Circle divided into 360 parts called degrees.

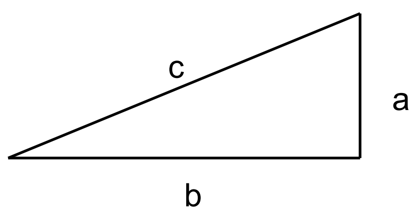
Minute (') 1/60 of a degree.
second (") 1/3600 of a degree.

Radian: $\theta = s/r$ where s is the arc length and r is the radius



$360° = 2\pi$ radians

soh cah toa



$\sec\theta = 1 / \cos\theta$
$\csc\theta = 1 / \sin\theta$
$\cot\theta = 1 / \tan\theta$

Pythagorean Theorem

$a^2 + b^2 = c^2$

For small angles, the length a is nearly equal to the arc length s and $\theta = s/c$ is nearly equal to the
$\sin\theta = a/c$.

$\sin\theta \sim \theta$ for small angles.

For small θ, c and b are nearly equal, so $\tan\theta = a/b$ is nearly equal to both θ and $\sin\theta$ for small values of θ.

VECTORS

Scalar - magnitude (i.e. volume, mass, speed)

Vector - magnitude and direction (i.e. force, velocity)

Quantities that are vectors are represented with an arrow over the symbol (i.e. $\vec{B}$) or the symbol is represented in bold (i.e. **B**). When drawn graphically, vectors are represented as arrows.

Properties of Vectors

Equality

A = **B** if $|A| = |B|$ and their directions are the same.

Addition

C = **A** + **B**

Negative of a vector

-**A** has the same magnitude but opposite direction as **A**.

Subtraction

C = **A** – **B** is the same as adding the negative of **B** to **A.** e.g. **C** = **A** + (**-B**)

Multiplication by a scalar

B = *s***A** has a magnitude $s|\mathbf{A}|$.
If *s* is positive, **B** = *s***A** has the same direction as **A**.
If *s* is negative, **B** = *s***A** has the opposite direction as **A**.

ADDING VECTORS

When two or more vectors are to be added, the following systematic procedure is recommended:

1. Select a coordinate system.

2. Draw a sketch of the vectors to be added (or subtracted), with a label on each vector.

3. Find the x and y components of all vectors.

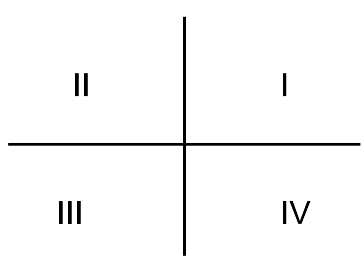
4. Find the algebraic sum of the components in both the x and y directions.

5. Use the Pythagorean Theorem to find the magnitude of the resultant vector.

6. Use a suitable trigonometric function to find the angle that the resultant vector makes with the x-axis. USE THE INVERSE TANGENT.

There is a slight complication in applying step 6. Suppose $\mathbf{A}_x = 2$ m and $\mathbf{A}_y = -2$m. Then, $\tan\theta = -1$. However, there are two angles for which $\tan\theta = -1$: $\theta = 135°$ and $\theta = 315°$.
Need to look at quadrant.

Quadrants



Rectangular Coordinates -
x and y coordinates.

Polar Coordinates -

R and θ: + is counterclockwise, while - is clockwise.

THE VELOCITY VECTOR

Position Vector *r* of point *P* is a vector that goes from the origin of the coordinate system to the point.

r = x**i** + y**j**
Δr = r – r₀

The **average velocity** between two points is:

$$\vec{v}_{av} = \frac{r_2 - r_1}{t_2 - t_1} = \frac{\Delta r}{\Delta t}$$

The **instantaneous velocity** at a point is defined as:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

$$v_x = \frac{dx}{dt}; v_y = \frac{dy}{dt}$$

$$\vec{v} = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

For motion in a plane:

$$|\vec{v}| = v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} \quad \text{and} \quad \tan\alpha = \frac{v_y}{v_x}$$

The **average acceleration** between two points is defined as:

$$\vec{a}_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

The **instantaneous acceleration** at a point is defined as:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$a_x = \frac{dv_x}{dt}; a_y = \frac{dv_y}{dt}; a_z = \frac{dv_z}{dt}$$

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

PROJECTILE MOTION

Projectile motion refers to the motion of an object in a plane (two dimension) only under the influence of gravity.

Following assumptions can be made to simplify problem:

1. The acceleration due to gravity, ***g***, varies from location to location, and is directed downward. In Austin, TX $g = 9.79\ m/s^2$. In Rochester, MN, $g = 9.80\ m/s^2$ In Salinas California it is ????
2. Effect of air resistance is negligible.
3. The rotation of the earth does not affect the motion.

Problem Solving Strategy:

1. Select the coordinate system (x and y axes) and define a + direction. Often, textbooks select + to be in the opposite direction of ***g***. However, you can choose + whichever direction you want.
2. Choose an origin. Usually the origin ($x = 0$ and $y = 0$) is chosen to be at the initial position of the object.
3. Resolve initial velocity vector into x and y components.
4. Treat horizontal and vertical motion independently.
5. Create a table of x and y values.
6. Choose equations based on what knowns and unknown you have.
   THE ONLY LINK BETWEEN *X* AND *Y* EQUATIONS IS **TIME.**

$$v_{xo} = v_o \cos\theta_0$$
$$v_{yo} = v_o \sin\theta_0$$
$$v_y = v_{yo} + gt$$
$$y - y_o = v_{yo}t + \frac{1}{2}gt^2$$
$$v_y^2 = v_{yo}^2 + 2g(y - y_o)$$

$$x - x_o = v_{xo}t$$

Range equation:

$$R = \frac{v_o^2 \sin 2\alpha_o}{g}$$

UNIFORM CIRCULAR MOTION

$$a_{rad} = \frac{v^2}{R}$$

$$v = \frac{2\pi R}{T}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2}$$

Derivation

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i}$$

By similar triangles:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}\ ;\ a = \frac{v\Delta r}{r\Delta t}\ ;\ a_r = \frac{v^2}{r}$$

NEWTON'S THREE LAWS OF MOTION

Inertia - property of matter that describes its opposition to change in motion.

Mass - measure of inertia.

Units for Mass: *kg, slugs (lb)   - 1kg = 2.2 lb*

A **Frame of Reference** is an extended object whose parts are at rest relative to each other. If no forces act on an object, any frame of reference with respect to which the acceleration of the object remains zero is an inertial frame of reference.

Force - push or pull. Something capable of changing an object's motion.

LAW 1 - LAW OF INERTIA

An object at rest will remain at rest and an object in motion will continue in motion with a constant velocity (constant speed in a straight line) unless it experiences a net external force (called the resultant force).

LAW 2 -

The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the resultant force.

$\boldsymbol{a} = \boldsymbol{F}/m$

$\Sigma \boldsymbol{F} = m\boldsymbol{a}$ If $\Sigma \boldsymbol{F} = 0$, then $\boldsymbol{a} = 0$

Units for force:

$1N = 1kg \cdot m/s^2$;

$1dyne = 1g \cdot cm/s^2$

Pound vs. slug   *1 lb = 4.45 N   1 kg weighs 2.2 lbs*

weight- gravitational force, $\boldsymbol{w} = m\boldsymbol{g}$

LAW 3 - LAW OF ACTION AND REACTION

For every action there is an equal and opposite reaction: $F_B = -F_A$

or

If two bodies interact, the force exerted on body 1 by body 2 is equal in magnitude and opposite in direction to the force exerted by body 2 on body 1.

If A exerts a force F on B, then B exerts an equal and opposite force F on A.

If A pushes B with a force of *3 N*, then B pushes A with a force of *3 N*.

If the earth pulls on the ball with a force of 6 N, then the ball pulls on the earth with a force of *6N*.

If the swimmer pushes backwards on the water with a force of *4 N*, then the water pushes the swimmer forward with a force of *4 N*.

MORE ON FORCES

Fundamental Forces

1. The gravitational force
2. The electromagnetic force
3. The strong nuclear force - holds nucleons together. Nucleons are protons and neutrons.
4. Weak nuclear force - force responsible for a special kind of nuclear decay called beta decay.

Newton's Law of Gravity

$\vec{F}_{1,2} = -G\dfrac{m_1 m}{r_{1,2}{}^2}\hat{r}_{1,2}$ where $G = 6.67 \times 10^{-11}$ N•m^2 /kg^2

G is known as the **universal gravitational constant.**

$g = GM_E/(R_E)^2 \approx 9.80\ m/s^2$ near the surface of the earth.

Action-at-a-distance- space altered; change propagated through space at the speed of light.
Field theory.

Contact forces

Result of molecular forces.
Complicated manifestations of basic electromagnetic forces.

1. Hooke's Law - Force proportional to displacement
   It is a restoring force because it tends to restore spring (or material) to its initial configuration.

   $\boldsymbol{F} = -k\boldsymbol{x}$ where k is the spring constant and measures stiffness of spring or material.

2. Normal force, F_N - perpendicular force between two bodies in contact with each other due to resistance to compression. For example, if you push against a wall, the force of the wall on you is called a normal force.

FRICTION

Friction: the resistance to relative motion between two surfaces in contact. Due to bonding of molecules of the two surfaces. Manifestation of electro-magnetic force. It opposes the direction of the applied force.

Experimentally it has been found:

1. The force of static friction between any two surfaces is opposite the direction of impending motion F_N and can have values: $f_s \leq \mu_s F_N$ where μ_s is called the coefficient of static friction and F_N is the normal force.

2. The force of kinetic friction is opposite the direction of motion, and is given by $f_k = \mu_k F_N$ where μ_k is the coefficient of kinetic friction and F_N is the normal force.

3. The values μ_s and μ_k depend on the nature of the surfaces, but μ_k is generally less than μ_s. μ_s varies between *0.01* for smooth surfaces to *1.5* for rough surfaces.

Drag forces: the resistance that occurs when an object moves through a fluid that tends to decrease the speed of the object. Depends on shape of object, properties of fluid and speed of object.

For small speeds, the drag force is proportional to the speed of the object; for large speeds, the drag force is proportional to the square of the speed.

$$\sum F_y = mg - bv^n = ma$$

At terminal speed, $a = 0$

$$bv_t^n = mg$$

$$v_t = \left(\frac{mg}{b}\right)^{\frac{1}{n}}$$

FREE-BODY DIAGRAMS AND APPLICATIONS OF NEWTON'S LAWS

Free-body diagram: when you isolate an object and show all of the forces acting on the object.

Steps for Solving Free-Body Diagrams:

1. Draw a neat diagram of the object of interest.
2. Label forces:
   a) Tension: always drawn out in direction of string or cable
   b) Weight: $\boldsymbol{w} = m\boldsymbol{g}$ **-** draw straight down.
   c) Normal force: perpendicular to surface
   d) Friction: opposite motion or impeding motion
3. Label mass
4. Label acceleration
5. Label x and y axes, with the x axis in the direction of the acceleration
6. Choose + and - directions for each axis.
7. Break forces into x and y components
8. Use:

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$

If there is friction, you can use another equation:

$$f_s \leq \mu_s$$
$$f_k = \mu_k$$

9. Solve

UNIFORM CIRCULAR MOTION

$$a_{rad} = \frac{v^2}{R}$$

$$v = \frac{2\pi R}{T}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2}$$

Derivation

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i}$$

By similar triangles:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}\ ;\ a = \frac{v\Delta r}{r\Delta t}\ ;\ a_r = \frac{v^2}{r}$$

The tangential acceleration causes the change in speed of the particle.

$$a_t = \frac{d|v|}{dt}$$

The radial acceleration arises from the change in direction of the velocity.

$$a_r = v^2 / r$$

$$a = \sqrt{\left(a_r^2 + a_t^2\right)}$$

$$\boldsymbol{a} = a_t\ \hat{\boldsymbol{\theta}} + a_r\,\hat{\boldsymbol{r}} = \frac{d|v|}{dt}\hat{\theta} + \frac{v^2}{r}\hat{r}$$

WORK AND POWER

Work: the work done by a constant force is defined as the product of the component of the force along the direction of the displacement and the magnitude of the displacement.

Work is a scalar, even if it is the product of two vectors.

$W = F_x\Delta x = F\cos\theta\ \Delta x$

$W_{total} = F_{1x}\Delta x + F_{2x}\Delta x + F_{3x}\Delta x + \ldots = (F_{1x} + F_{2x} + F_{3x})\Delta x = F_{net}\Delta x$

Work of a variable force in one dimension is equal to the area under a force vs. displacement graph.

$W = \lim \Delta x \to 0 \sum F_x dx =$

$$W = \int_{x_1}^{x_2} F_x dx$$

Units of work:
$N \cdot m =$ *joule (J),*
foot • lb,
$\text{dyne} \cdot \text{cm} =$ *erg*

Dot Product

$\vec{A} \bullet \vec{B} = AB\cos\theta$ _ Definition

$\vec{A} \bullet \vec{A} = A^2$

$(\vec{A} + \vec{B}) \bullet \vec{C} = \vec{A} \bullet \vec{C} + \vec{B} \bullet \vec{C}$

$A \bullet B = (A_X\hat{i} + A_y\hat{j} + A_Z\hat{k}) \bullet (B_X\hat{i} + B_Y\hat{j} + B_Z\hat{k}) =$

$= A_X B_X + A_Y B_Y + A_Z B_Z$

Work in three dimensions (Definition of Work) $W = \int_{s_1}^{s_2} \hat{F} \bullet d\hat{s}$

<u>Power:</u> time rate of doing work

$dW = \hat{F} \bullet d\hat{s} = \hat{F} \bullet v dt$

$$P = \frac{dW}{dt} = \hat{F} \bullet v$$

Units of Power: w*att (W), ft • lb/s, horsepower, hp = 550 ft • lb/s*

ENERGY

Energy: the ability to do work

Units of Energy: the same as those of work. *joule (J), foot • lb, erg*
Energy is a scalar

Kinetic energy, *K*: energy due to motion

$$K = \frac{1}{2}mv^2$$

Work Energy Theorem

$W_{total} = \Delta K = ½\, m(v_f)^2 – ½\, m(v_i)^2$

$$W = \int F_s\, ds$$

$$F_s = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

$$W_{total} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} m\frac{dv}{dt}ds = \int_{v_1}^{v_2} mv\,dv$$

$$W_{total} = \int_{s_1}^{s_2} F_s\, ds = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

CONSERVATIVE VS NON-CONSERVATIVE FORCES

Conservative Force:
The work done by a conservative force has the following properties:
1. It can always be expressed as the difference between the initial and final values of a *potential energy* function.
2. It is independent of the path of the body and depends only on the starting and ending points.
3. When starting and ending points are the same, the total work is zero.

Conservative forces include the gravitational force, the elastic force and the electric force.

$$W_c = \int_{x_i}^{x_f} \hat{F}_x \bullet d\hat{s} = -\Delta U = U_i - U_f \quad \text{(Definition)}$$

Potential Energy, *U*: energy due to position or stored energy.

Gravitational Potential Energy, U= mgy
Elastic potential energy = $\frac{1}{2}kx^2$

Gravitational potential energy near the surface of the Earth

$dU = -\mathbf{F}\bullet d\mathbf{s} = -(-mg\mathbf{j})\bullet(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) = +mgdy$

$U = \int mgdy = mgy + U_0$

$U = mgy + U_0$ *where* U_0 is an arbitrary constant of integration.

In one dimension, a conservative force is equal to negative the derivative of the potential energy function.
$dU = -F_x\, dx$ (one dimension)

$F_x = -dU/dx$

Exercise: Find the potential energy of a spring, where *F* = -*kx*

EQUILIBRIUM

A particle is in **equilibrium** if the net force on the particle is zero.

A particle is in **stable equilibrium if** a small displacement results in a restoring force that accelerates the particle back to the equilibrium position.
The equilibrium is stable where the potential-energy function is a minimum (2^{nd} derivative of potential energy function is positive).

A particle is in **unstable equilibrium** if a small displacement results in a force that accelerates the particle away from its equilibrium position.
The equilibrium is unstable where the potential-energy function is a maximum (2^{nd} derivative of potential energy function is negative).

A particle is in **neutral** equilibrium if a small displacement results in zero force and the particle remains in equilibrium. Occurs where the force is zero at $x = 0$ an neighboring points (or where 1^{st} derivative of potential-energy function is zero a $x = 0$ and at neighboring points).

MORE ON ENERGY

Law of Conservation of Energy: energy can never be created nor destroyed. Energy may be transformed from one form to another, but the total energy of an isolated system is always constant. Total energy of the universe is constant.

$E_{in} - E_{out} = \Delta E_{sys}$

where $E_{syst} = E_{mech} + E_{therm} + E_{chem.} + E_{other}$

Mechanical Energy, $E_{mech} = U_{sys} + K_{sys}$

Some notes regarding your text:
Your authors (Tipler and Mosca) consider the conservative forces to be internal forces.
Gravitational force: earth is part of the system in all of the problems.
Spring force: spring and what the spring is attached to are part of the system in all of the problems.

Frictional work: Your authors (Tipler and Mosca) state that $-f_k\,\Delta s$ is not the work done by friction on the sliding block, because "the actual displacement of the kinetic frictional force on a block is not in general equal to the displacement of the block. However, it can be shown that f Δs is equal does equal the increase in thermal energy of the block-table system" Tipler, Mosca, Physics for Scientists and Engineers, 5th edition, p. 195. $\Delta E_{therm} = f\,\Delta s$

Work total = $W_{nc\,(external)} + W_{nc\,(internal)} + W_c(_{internal})$
$W_c(_{internal}) = -\Delta U$

$\Delta E_{mech} = \Delta U + \Delta K_{sys}$

$W_{nc\,(external)} + W_{nc\,(internal)} = \Delta E_{syst} = \Delta E_{mech} + \Delta E_{therm} + \Delta E_{chem.} + \Delta E_{other}$

Conservation of Mechanical Energy:
If there is no heat loss or friction, mechanical energy is conserved.

If $W_{ext} = 0$, and friction is negligible, then $E_{mech\,f} = E_{mech\,i}$
$U_i + K_i = K_f + U_f$ (Conservation of mechanical energy)

Efficiency: (useful work out / work in) = (useful energy out/energy in)

Unusable energy is lost into heat with each transformation. Heat is graveyard of useful energy. No machine is 100 % efficient.

<u>Work-Energy Theorem</u> for special case when dealing with just mechanical energy, friction and external (non-conservative) forces.

$W_{nc\ (external)} = W_{ext} = \Delta E_{mech} + f_k\,\Delta s$

$$**** \; W_{ext} = \Delta K_{sys} + \Delta U_{sys} + f_k\,\Delta s \; ****$$

RELATIVE VELOCITY AND RELATIVE ACCELERATION

Let $\boldsymbol{v_{pA}}$ be the velocity of a particle *p* relative to a coordinate system *A*.
(i.e. the velocity of a boat, *p*, relative to river, *A)*.

Let $\boldsymbol{v_{AB}}$ be the velocity of system *A* relative to system *B*.
(i.e. the velocity of river ,*A*, relative to the shore, *B*).

Let v_{pB} be the velocity of particle *p* relative to system *B*.
(i.e. the velocity of the boat , *p,* relative to the shore, *B*)

Then. $\boldsymbol{v_{pB}} = \boldsymbol{v_{pA}} + \boldsymbol{v_{pB}}$

The acceleration of a particle measured by an observer in the Earth's frame of reference is the same as that measured by any other observer moving with constant velocity relative to the Earth's frame.

$\mathbf{r} = \mathbf{r'} + \mathbf{v_0}t$

$\mathbf{r'} = \mathbf{r} - \mathbf{v_0}t$

$$\frac{dr'}{dt} = \frac{dr}{dt} - v_0$$

$\mathbf{v'} = \mathbf{v} - \mathbf{v_0}$

$$\frac{dv'}{dt} = \frac{dv}{dt} - \frac{dv_0}{dt}$$

CENTER OF MASS

Center of Mass: point in which all the mass seems to be concentrated.

$$\vec{x}_{cm} = \frac{\sum_i m_i x_i}{M}, y_{cm} = \frac{\sum_i m_i y_i}{M}$$
$$\text{where\_} M = \sum_i m_i$$

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{r}_{cm} = \int \vec{r}dm$$

The center of mass of a system moves like a *particle* of mass $M = \Sigma\, m_i$ under the influence of the net external force acting on the system.

The net external force acting on a *system* equals the total mass of the system multiplied by the acceleration of the center of mass.

$$F_{ext} = Ma_c$$

$$\vec{r}_{cm} = \frac{\sum m_i r_i}{\sum m_i}$$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i} = \frac{\sum \vec{F}_i}{M} = \frac{\sum \vec{F}_{i,ext} + \sum \vec{F}_{i,int}}{M}$$

$$\vec{F}_{NET} = \sum_i F_{i,ext} = Ma_{cm}$$

If the net external force on a system of objects is zero, then $a_c = 0$.

MOMENTUM

Momentum: $\boldsymbol{p}=m\boldsymbol{v}$, $p_x = mv_x$; $p_y = mv_y$; $p_z = mv_z$

For a given particle:

$$\sum \boldsymbol{F} = m\vec{\boldsymbol{a}} = m\frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}(m\vec{\boldsymbol{v}}) = \frac{d\boldsymbol{p}}{dt}$$

The vector sum of the forces acting on a particle equals the time rate of change of momentum of the particle.

In a system of particles,

$$M\frac{dr_{cm}}{dt} = m_1\frac{dr_1}{dt} + m_2\frac{dr_2}{dt} =$$
$$= \sum m_i \frac{d\vec{r}_i}{dt} = \sum m_i \vec{v}_i = M\vec{v}_{cm}$$

$$\vec{P}_{sys} = \sum_i m_i \vec{v}_i = \sum_i \vec{P}_i = M\vec{v}_{cm}$$
$$\frac{dP_{sys}}{dt} = M\frac{dv_{cm}}{dt} = Ma_{cm} = F_{net}$$

Law of Conservation of Momentum
If the net external force on a system is zero, the total momentum of the system remains constant.

$\boldsymbol{F}_{net}$, $= d\boldsymbol{P}_{sys}/dt$ If $\boldsymbol{F}_{net} = 0$, *then* $\boldsymbol{P}$ is constant.

Example with 2 particles.

$$P = p_1 + p_2 = \sum p_i$$

$$\frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{dp_2}{dt} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 0$$
$$F_1 = -F_2$$
$$\frac{dp_1}{dt} = -\frac{dp_2}{dt};\ \frac{d}{dt}(p_1 + p_2) = 0$$
$$p_1 + p_2 = constant$$

IMPULSE

Impulse, $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt$

$$dp = Fdt$$
$$\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt = \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\overline{F} = \frac{1}{\Delta t} \int \vec{F} dt$$

COLLISIONS

Elastic Collisions:
Collision in which both momentum is conserved and kinetic energy is conserved.

$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$ *Relative velocities in elastic collisions.*

$$u_i = -u_f$$

In elastic collisions, the speed of recession is equal the speed of approach.

Inelastic Collisions:
Collision in which momentum is conserved but kinetic energy is not.

Perfectly inelastic collisions: two objects are in an inelastic collision if the two objects stick together after the collision, so that their final velocities are the same.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Coefficient of Restitution, *e*, is the measure of the elasticity of a collision.

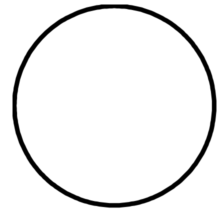
$$\boldsymbol{e} = \frac{|\boldsymbol{v}_{2f} - \boldsymbol{v}_{1f}|}{|\boldsymbol{v}_{2i} - \boldsymbol{v}_{1f}|} = \frac{\boldsymbol{v}_{rec}}{\boldsymbol{v}_{app}}$$

For elastic collisions, *e* = 1

For perfectly inelastic collision, *e* = 0

ROTATION

The **radian** is defined as 1 radian is the angle where *s = r; Δθ = s/r*



1 rad ≈ 57.3°

Angular displacement: $\Delta\theta$

Average angular velocity: $\omega = \dfrac{\theta_f - \theta_i}{t_f - t_i} = \dfrac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity: $\omega = lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

Average angular acceleration: $\overline{\alpha} = \dfrac{\omega_f - \omega_i}{t_f - t_i} = \dfrac{\Delta\omega}{\Delta t}$

Instantaneous angular acceleration:

$$\alpha = lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

$$\alpha = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$

Rotational motion when **α = constant**

$$\omega = \omega_0 + \alpha t$$

$$\vec{\theta} - \vec{\theta}_0 = \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha} t^2$$

$$\omega^2 = \omega_0^2 + 2\vec{\alpha}(\vec{\theta} - \vec{\theta}_0)$$

$$\vec{\theta} - \vec{\theta}_0 = \frac{1}{2}(\vec{\omega}_0 + \vec{\omega})t$$

Straight line motion when **a = constant**

$$v = v_o + at$$

$$\vec{x} - \vec{x}_o = \vec{v}_o t + \frac{1}{2}\vec{a} t^2$$

$$\vec{x} - \vec{x}_o = \dot{v}_{avg} t = (\frac{\vec{v}_o + \vec{v}}{2})t$$

$$v^2 = v^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

Relations Between Angular and Linear Quantities

$$s = r\Delta\theta;$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$v = r\omega$$

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\boldsymbol{a} = \sqrt{\boldsymbol{a}_t^2 + \boldsymbol{a}_c^2}$$

Centripetal Force

$$F_c = ma_c = m\frac{v^2}{r}$$

Centripetal forces act toward the center of the circular path along which the object moves.

Kinetic Energy of Rotation

$$\frac{1}{2}m_i v_i^{\,2} = \frac{1}{2}m_i r_i^{\,2}\omega^2$$

$$K = \sum_i \frac{1}{2}m_i r_i^2 \omega^2$$

$$K = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

$$I = \sum_i m_i r_i^2 \Rightarrow I = \int r^2 dm$$

$$K = \frac{1}{2}I\omega^2$$

I is called **moment of inertia** because it represents resistance to change in motion.

TORQUE

The ability of a force to rotate a body about some axis is measured by a quantity called the **torque,** τ. The torque due to a force ***F*** has a magnitude given by the equation

$$\tau = Fl = Fr\sin\phi$$

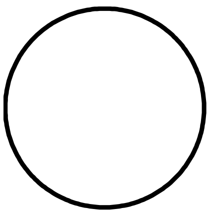
where l is called the **lever arm** (or moment arm) of the force ***F*.**

The **lever arm** l is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force.

The *net torque* on a body is found by summing up the individual torques.

Convention has the torque be + for counterclockwise rotation and - for clockwise rotation.

Units of torque: $N \cdot m$ or $lb \cdot ft$.



$F_t = ma_t$, $F_t r = mra_t$, $a_t = r\alpha$, $F_t r = mr^2\alpha = \tau$

Torque of a Rotating Object:

$$\Sigma\tau = \left(\sum_i m_i r_i^2\right)\alpha = I\alpha$$

$I = \sum_i m_i r_i^2$ For a system of particles

For a continuous object, $\boldsymbol{I = \int r^2 dm}$

$$\Sigma\tau = I\alpha$$

The angular acceleration of an object is proportional to the net torque acting on it. The proportionality constant, I, between the net torque and angular acceleration is the moment of inertia.

Parallel-Axis Theorem

$$I = I_{cm} + Mh$$

ENERGY AND WORK OF A RIGID BODY

Work and power in rotational Motion

$$dW = F_{tan} ds = F_{tan} R d\theta = \tau d\theta$$

If the torque is constant:

$$\int_{\theta_1}^{\theta_2} \tau d\theta = \tau(\theta_2 - \theta_1) = \tau\Delta\theta$$

When a torque does work on a rotating body, the kinetic energy changes by an amount equal to the work done:

$$\tau d\theta = (I\alpha I\alpha) = I\frac{d\omega}{dt} d\theta = I\frac{d\theta}{dt} d\omega = I\omega\omega d$$

$$W = \int_{\omega_1}^{\omega_2} I\omega\omega d = \frac{1}{2}I\omega_2{}^2 - \frac{1}{2}I\omega_1{}^2$$

The power associated with work done by a torque:

$$\frac{dW}{dt} = \tau\frac{d\theta}{dt}$$

$$P = \tau\omega$$

Rolling Objects (without slipping)

$s = R\phi$

$v_{cm} = ds/dt = R\, d\phi/dt$ or $v_{cm} = R\omega$

$a_{cm} = R\alpha$

$$\frac{1}{2}Mv_{cm}{}^2 + \frac{1}{2}I_{cm}\omega^2 = K$$

ANGULAR MOMENTUM
CROSS PRODUCT

Right Hand rule: When the fingers of the right hand curl in the direction of rotation, thumb points in direction of vector.

$\mathbf{A} \times \mathbf{B} = (AB \sin \varphi)\ \hat{\mathbf{n}}$

$\mathbf{A} \times \mathbf{A} = 0$

$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

Distributive law:

$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$

$\mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{j} \times \mathbf{i} = -\mathbf{k}$
$\mathbf{j} \times \mathbf{k} = \mathbf{i} \qquad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$
$\mathbf{k} \times \mathbf{i} = \mathbf{j} \qquad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$
$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$

$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$

Torque $\tau = r \times F$

ANGULAR MOMENTUM

The angular momentum ***L*** of a particle about the origin is defined as:

L = **r** x **p** = **r** x m**v**

L = mvrsinφ = mvl where *l* is the perpendicular distance from the line of *v* to *O*, the axis of rotation.
The direction of the angular momentum is given by the right hand rule of the vector product.

For Circular Motion:
The magnitude of the angular momentum of a particle *i* is then
$L_i = m_i (r_i \omega) r_i = m_i r_i^2 \omega$.

The total angular momentum of a body lying in the xy plane is:

$$L = \sum L_i = (\sum m_i r_i^2)\omega = I\omega$$

When a body rotates about an axis of symmetry, its angular momentum vector *L* has the direction of the symmetry axis. When the axis of rotation is not the symmetry axis, the angular momentum is in general not parallel to the axis.
The net external torque acting on a system equals the rate of change of the angular momentum of the system.

$$\frac{dL}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{dr}{dt} \times \vec{p} + \vec{r} \times \frac{dp}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \frac{dp}{dt} = \vec{r} \times \frac{dp}{dt}$$

$$\vec{\tau}_{net} = \vec{r} \times \vec{F}_{net} = \vec{r} \times \frac{d\vec{p}}{dt} = \frac{d\vec{L}}{dt}$$

Conservation of angular momentum:
When the net external torque is equal to zero, then the total angular momentum of a system is constant.

$$\frac{dL}{dt} = 0 \qquad L = \text{constant or} \quad I_i\omega_i = I_f\omega_f$$

The angular momentum of a system is conserved when the net external torque acting on the system is zero.

STATIC EQUILIBIRUM

Conditions for equilibrium:

First condition of equilibrium:
$\Sigma \boldsymbol{F} = 0$

Second condition of equilibrium:
$\Sigma \boldsymbol{\tau} = 0$

PROBLEM SOLVING STRATEGY
1. Draw diagram of system
2. Draw free-body diagram of objects
3. Establish coordinates
4. $\Sigma \boldsymbol{F} = 0$
5. Choose origin for calculating torque. $\Sigma \boldsymbol{\tau} = 0$
6. Solve equations

Center of Gravity: point in which all the weight seems to be concentrated.

Center of Mass: point in which all the mass seems to be concentrated.

$$X_{cg} = \frac{\sum w_i x_i}{W}$$

$$X_{cg} = \frac{\sum m_i g x_i}{gM} = \frac{\sum m_i y_i}{m_i} = X_{cm}$$

In a uniform gravitational field, the center of gravity is the same as the center of mass.

The net external force acting on a system equals the total mass of the system multiplied by the acceleration of the center of mass.

$F_{ext} = Ma_c$

If the net external force on a system of objects is zero, then $a_c = 0$.

$\boldsymbol{\tau} = \boldsymbol{r}_{cg} \times \boldsymbol{W}$

DENSITY AND PRESSURE

Density, $\rho =$ *mass / volume*
Measures the compactness of a substance.

Specific Gravity: density of a substance / density of water
density relative to that of water.

Pressure: force / area

$$P = \frac{dF}{dA}$$

Units:

1 Pascal (Pa) = 1 N/m^2 = 14.7 lb/in^2
1 atm = 1.013 bar = 1013 millibar = 76 cm Hg
1 torr = 1 mmHg

Absolute Pressure: actual pressure
Gauge Pressure: pressure of fluid
Atmospheric Pressure: pressure of atmosphere

Variations of Pressure with Depth

The absolute pressure, ***P***, at a depth *h* below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount *dgh.* Moreover, the pressure is not affected by the shape of the vessel.

$$PA - P_0A - \rho A\Delta hg = 0$$

Bulk Modulus:

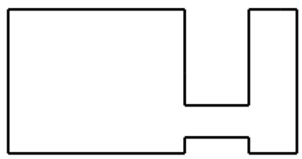
$$B = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{-\Delta V/V} = \frac{\Delta P}{-\Delta V/V} = -\Delta PV/\Delta V$$

Pascal's Principle: the pressure applied to an enclosed liquid is transmitted undiminished to every point in the fluid and to the walls of the container. Bed sores, cerebrospinal fluid (brain tumor), unborn baby, eye.

$P_1 = P_2$

$F_1 / A_1 = F_2 / A_2$

$F_2 = 1000 lbs \quad F_1 = 10 lbs$



$A_2 = 100 in^2 \qquad A_1 = 1 in^2$

Hydrostatic Paradox:

ARCHIMEDE'S PRINCIPLE

Buoyancy and Archimedes's Principle: Any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.

$\boldsymbol{F_b = W_f}$ where F_b = weight of object in air - "weight" of object in fluid.

A completely submerged object always displaces a volume of liquid equal to the object's own weight.

It makes no difference how deep the object is, because it is the difference in depth that counts.

The buoyant force is a result of the difference of pressure on an object due to it having different parts at different depths.

$$
\begin{aligned}
&F_B = F_2 - F_1 = P_2 A - P_1 A = \\
&d_f g h_2 A - d_f g h_1 A = d_f g V = \\
&m_f g = w_f
\end{aligned}
$$

Specific gravity = weight of object in air / weight of equal volume of water = w_0 / w=
= weight of object in air / weight loss when submerged in water = w_0 / w_{loss}

Hydrometer: Instrument used to measure the specific gravity of an object. If the specific gravity of a liquid increases, the hydrometer will float higher because less volume must be displaced to balance the forces.

BERNOULLI'S EQUATION

Ideal Fluid

1. Fluid is non-viscous: there are no internal frictional forces between adjacent layers.

2. Fluid is incompressible: density is constant.

3. Fluid motion is steady: velocity, density and pressure at each point in the fluid does not change with time.

4. Fluid moves without turbulence: zero angular velocity about center, or no eddy currents (irregular motion of fluid).

Flow line: the path an individual particle in a moving fluid. If the overall flow pattern does not change with time, the flow is called **steady flow.**

A **streamline** is a curve whose tangent at any point is in the direction of the fluid velocity at that point.

Turbulent flow: chaotic flow.

Equation of Continuity:
In steady flow, $dm_1 = dm_2 = \rho \Delta V$

$$\boldsymbol{\rho A_1 v_1 dt = \rho A_2 v_2 dt}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

Volume Flow Rate: ($dV/dt = Av$)

The mass flow rate: $\frac{dm}{dt} = \rho \frac{dV}{dt}$

If the fluid is compressible,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

BERNOULLI'S EQUATION

Bernoulli's Equation: (Conservation of energy equation)
Bernoulli's equation says that the sum of the pressure *P*, the kinetic energy per unit volume ($\frac{1}{2}dv^2$) and the potential energy per unit volume (*dgh*) has the same value at all points along a streamline.

<u>Derivation of Bernoulli's Equation</u>

In deriving Bernoulli's equation, we shall assume that the fluid is incompressible and non-viscous and flows in a non-turbulent, steady state manner. Consider the flow through a non-uniform pipe in a time Δt, as illustrated in the figure at right. The force on the lower end of the fluid is P_1A_1, where P_1 is the pressure at the lower end. The work done on the lower end of the fluid by the fluid behind it is

$$W_1 = F_1\, dx_1 = P_1 A_1\, dx_1 = P_1 V$$

where *V* is the volume of the lower region in the figure. In a similar manner, the work done on the fluid on the upper portion in the time Δt is

$$W_2 = -P_2 A_2\, dx_2 = -P_2 V$$

(Note that the volume of fluid that passes through A_1 in a time Δt equals the volume that passes through A_2 in the same time interval.) The work W_2 is negative because the force on the fluid at the top is opposite its displacement. Thus the net work done by these forces in the time Δt is

$$W = P_1 V - P_2 V$$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing its gravitational potential energy. If *m* is the mass passing through the pipe in the time interval Δt, then the change in kinetic energy of the volume of fluid is

$$dK = \tfrac{1}{2}\, \rho\, dV(\, v_2^2 - v_2^2)$$

The change in its potential energy is $dU = \rho\, dV\, g(y_2 - y_1)$

We can apply the work-energy theorem in the form $W = dK + dU$ to this volume of fluid to give

$(P_1 - P_2)\, dV = \frac{1}{2}\rho dV(v_2^2 - v_1^2) + \rho dVg(y_2 - y_1) = P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$

Let us move those terms that refer to point 1 to one side of the equation and those that refer to point 2 to the other side:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \textit{(Equation 1)}$$

This is **Bernoulli's equation**. It is often expressed as

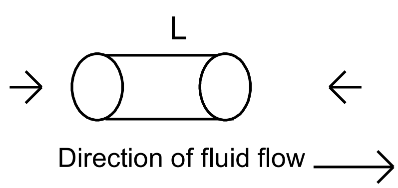
$$P + 1/2\, \rho v^2 + \rho g y = \textit{constant} \quad \textbf{(Equation 2)}$$

Bernoulli's equation states that the sum of the pressure (P), the kinetic energy per unit volume ($1/2\, \rho v^2$), and potential energy per unit volume ($\rho g y$), has the same value at all points along a streamline.

When a fluid flows through a tube there will be a pressure drop because of friction.

Pressure Gradient: pressure drop per unit length.

Pressure gradient = $\frac{P_1 - P_2}{L}$



Poiseuille's Law

Flow Rate: dV/dt =Volume /time = $\frac{\boldsymbol{P_1 - P_2}}{\boldsymbol{R}}$

$$\boldsymbol{R} = \frac{\boldsymbol{P_1 - P_2}}{\frac{8\eta \boldsymbol{L}}{\pi \boldsymbol{r}^4}} = (\boldsymbol{P_1} - \boldsymbol{P_2})\left(\frac{\pi \boldsymbol{r}^4}{8\eta \boldsymbol{L}}\right)$$

where η = viscosity, L = length, r = inside radius and P_1 and P_2 = pressures.

$$K = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\vec{v}_i \bullet \vec{v}_i)$$

$$\vec{v}_i = \vec{v}_{cm} + \vec{u}_i$$

$$K = \sum_i \frac{1}{2} m_i (\vec{v}_{cm} + \vec{u}_i) \bullet (\vec{v}_{cm} + \vec{u}_i) = \sum_i \frac{1}{2} m_i v_{cm}^2 + \sum_i \frac{1}{2} m_i u_i^2 + \vec{v}_{cm} \bullet \sum_i m_i \vec{u}_i =$$

$$= \sum_i \frac{1}{2} m_i v_{cm}^2 + \sum_i \frac{1}{2} m_i u_i^2$$

MECHANICAL PROPERTIES OF MATERIALS

Elastic: if object returns to its original shape when the forces a removed

Stress: Force/area

Two types of stress: longitudinal or normal stress and transverse or shear stress

Tensile vs. compressional

Translational vs. torsional

Strain: relative change in the dimension(s) or shape of a body

Tensile Strain: change in length /original length = $\frac{\Delta L}{L_o} = \frac{L - L_0}{L_0}$

Elastic Limit: maximum stress a material can experience without becoming permanently deformed.

Stress is proportional to strain

Elastic Modulus (*E*) *= stress/strain*

Young's Modulus *(Y) = longitudinal stess / longitudinal strain*

$$Y = \frac{F_n L_0}{A \Delta L}$$

Shear Modulus *(M)* = $\frac{F_t / A}{d / l}$

Bulk Modulus (B) = $\frac{-\Delta P}{\Delta V / V}$

GRAVITY

Ellipse: **locus of points whose sum of distances from the foci is constant.**

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Kepler's Laws of Motion

Law 1 - **All planets move in elliptical orbits with the sun at one focus.**

Law 2 - **A line joining any planet to the sun sweeps equal areas in equal times.**

Law 3 – **The square of the period of any planet is proportional to the cube of the semi-major axis of its orbit.** $\boldsymbol{T^2 = C\,r^3}$. $\boldsymbol{T^2 =(4\pi^2/GM^s{}_s)\,r^3}$

$1\ AU = 1.50 \times 10^{11}\ m = 9.30 \times 10^{16}\ mi$

Newton's Law of Gravity

$\vec{F}_{1,2}=-G\frac{m_1 m}{r_{1,2}{}^2}\hat{r}_{1,2}$ where $G = 6.67 \times 10^{-11}$ N•m^2 /kg^2

G is known as the **universal gravitational constant.**

$g = GM_E/(R_E)^2 \approx 9.80\ m/s^2$ near the surface of the earth.

Gravitational Potential Energy

Definiton: $dU = -\,\boldsymbol{F\bullet ds = -F_r dr = -(-GM_E m/r^2)dr = +(GM_E m/r^2)dr}$

$\boldsymbol{U = -(GM_E m)/r + U_0}$ $\quad \boldsymbol{U_0 = 0}$ at $r = \infty$

Cavendish: did experiments to find G.

Gravitational Field, g

$$\vec{F} = -G\frac{m_1 m}{r_{1,2}^{\;2}}\hat{r}_{1,2}$$

The gravitational field ***g*** is defined as ***g***= ***F/m*** where *m* is a small test charge.

The gravitational field at a point due a set of point masses is defined as ***g*** = Σ ***g***

The gravitational field at a point due to a continuous object is defined as $\boldsymbol{g} = \int d\boldsymbol{g}$

The gravitational field of the earth at $r \geq R_E$ **points towards the center of the earth and is given as** $\vec{g} = -G\frac{M_E}{r^2}\hat{r}$

Escape Speed

Escape speed is the minimal speed required for an object to escape a planet, moon etc.

E = K + U = 0 (at surface of the earth)

Total mechanical energy at infinity is zero, and total mechanical energy is conserved.

$$\frac{1}{2}mv_e^2 - \frac{GM_E m}{R_E} = 0$$

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} \approx 11.2 km/s$$

OSCILLATIONS

Periodic Motion: motion that repeats itself in a regular cycle.

Hooke's Law:

$F_s = -kx$; $a = \dfrac{F}{m} = -\dfrac{k}{m}x$

Simple Harmonic Motion: motion that can be described by a sine or cosine function. Occurs when the acceleration is proportional to the displacement and in the opposite direction of the displacement.
Period and frequency are independent on amplitude.

Amplitude, A: maximum distance that an object moves away from its equilibrium position. In the absence of friction, an object will continue in simple harmonic motion and reach a maximum displacement equal to the amplitude on each side of the equilibrium position.

Period, T: the time it takes the object to execute one complete cycle of motion.

Frequency, f: the number of cycles or vibrations per unit time.

Angular Frequency, ω: $\omega = 2\pi f = 2\pi/T$

Equations of Simple Harmonic Motion

$F_s = -kx = ma = m\, d^2x/dt^2$

$a = -(k/m)x$

Solution from ODE:

$x = A\cos(\omega t + \delta)$

$v = dx/dt = -\omega A\sin(\omega t + \delta)$

$a = dv/dt = -\omega^2 A\cos(\omega t + \delta) = -\omega^2 x$

Now, $a = -(k/m)x$ so $\omega = \sqrt{\dfrac{k}{m}} = 2\pi/T$ and $f = 1/T = \omega/2\pi$

$x_0 = A\cos\phi$ and $v_0 = -A\omega\sin\delta$

For a vertical spring, $y' = A\cos(\omega t + \delta)$ where y' is the vertical distance from the new equilibrium position.

For vertical spring: $U_{total} = \frac{1}{2}k(y')^2 + U_0$ where y' is measured from the new equilibrium position and U_0 is the total potential energy at $y' = 0$.

Elastic Potential Energy, energy stored in a stretched or compressed spring or other elastic material.

$U_s = 1/2kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \delta)$

$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \delta)$

$\omega^2 = k/m$

$K = \frac{1}{2}mv^2 = \frac{1}{2}kA^2\sin^2(\omega t + \delta)$

$E_{total} = U_s + K =$

$= \frac{1}{2}kA^2\cos^2(\omega t + \delta) + \frac{1}{2}kA^2\sin^2(\omega t + \delta) = \frac{1}{2}kA^2$

$$W_{nc} = (KE + PE_g + PE_s)_f - (KE + PE_g + PE_s)_i$$

The Simple Pendulum

$\Sigma F_t = -mg\sin\phi$ For small ϕ, $\sin\phi$ is close to ϕ.

$s = L\phi$.

$$\Sigma F_t = -mg\phi = m\frac{d^2s}{dt^2} = mL\frac{d^2\phi}{dt^2}$$

$$\frac{d^2\phi}{dt^2} = -\frac{g}{L}\phi = -\omega^2\phi$$

$$\phi = \phi_0\cos(\omega t + \delta)$$

$$\omega = \sqrt{\frac{g}{L}};\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}};T = 2\pi\sqrt{\frac{L}{g}}$$

Physical Pendulum

$$\tau = -(mgD)\theta = I\alpha = I\frac{d^2\theta}{dt^2};$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgD}{I}\sin\theta;$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgD}{I}\theta \rightarrow \theta_{small}$$

$$\omega = \sqrt{\frac{mgD}{I}};T = 2\pi\sqrt{\frac{I}{mgD}}$$

DAMPED OSCILLATIONS

$\boldsymbol{F_d} = -b\boldsymbol{v}$

$F_x = ma_x$

$-kx - b(dx/dt) = m(d^2x/dt^2)$

$x = A_0\, e^{-(b/2\tau)}\cos(\omega' t + \phi)$ where $\tau = m/b$ is called the *time constant.*

$\omega' = [\omega_0^2 - (b/2m)^2]^{1/2}$

$A = A_0\, e^{-(b/2\tau)}$

$b_c = 2m\omega_0$

When $b = b_c$, the system does not oscillate and is *critically damped*.
When $b > b_c$, the system is *over damped.*

Quality Factor, $Q = \omega_0\tau = 2\pi / |\Delta E| / E_{cycle}$

RESONANCE

The amplitude and therefore the energy of a system in the steady state condition depend not only on the amplitude of the driver, but also on its frequency.

Natural Frequency of an oscillator is the frequency of that oscillator when no driving or damping forces are present.

Resonance: if the driving frequency is equal to the natural frequency of the system, the system will oscillate with amplitude much larger than the driving force.

Resonant Frequency: natural frequency of a system.

Examples of Resonance: